ASYMPTOTIC STAGE OF GROWTH OF A VAPOR BUBBLE

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UDC 536.423

The asymptotic stage of growth of bubbles which form at a surface during the boiling of a saturated liquid is determined by inertial forces. The calculated dependence of the bubble radius on time obtained is confirmed by experimental data.

The idealization of the problem of vapor bubble dynamics (like any conjugate problem), consisting in the substitution of the proper system of boundary conditions for a complete joint examination of the interacting boundary phases, assumes the selection of a causal diagram of the phenomenon. This most important step of the analysis is often omitted either under the influence of traditional approaches to the problem or because of the apparent obviousness.

The relatively simple example of the asymptotic stage of growth of a vapor bubble in a saturated liquid is examined below, analyzed from the positions of two different causal diagrams.

The first solutions of the problem of bubble growth in a uniformly heated liquid are widely known [1, 2].

The assumption that the growth of the bubble is limited by heat exchange between the heated liquid and the bubble is fundamental in the solution of this problem. Actually, the single constant factor in this case is the value $\Delta T = T_L - T_V$ (with the growth of the bubble T_V approaches the constant value T_S). Then one can assume that the asymptotic stage of growth, when the initial disturbances have relaxed, will be determined by this constant factor. The problem is reduced to the problem of heat conduction in a liquid with the boundary condition $\partial T/\partial R = (\rho_V L/\lambda)\dot{R}$ at the surface of the bubble (the causal diagram of Boshnyakovich).

The experiments of Dergarabedyan [3] fully confirmed this analysis.

The other classical case is bubble growth from a superheated surface into a liquid saturated in bulk. The experimental results of Cole and Shulman [4] are presented in Fig. 2. The theoretical studies of this case of bubble dynamics [5, 6] follow the causal diagram of the works mentioned above [1, 2] and the analytical results naturally prove to be almost identical. The law of bubble growth has the form [5]

$$R = \frac{4}{3\sqrt{\pi}} \mathrm{Ja} \sqrt{a_L \tau}. \qquad (1)$$

In this case, however, the agreement of theory and experiment cannot be considered successful.

To begin with, equations of the type (1) contain only one parameter of the process, the Jacob number (the initial superheat), and while differing in the values of the numerical coefficients they always yield a unique solution $R(\tau)$ for a given value of Ja. At the same time the experimental data always have a large separation at identical values of Ja, which was indicated earlier in [7]. Apparently, satisfactory agreement with experiment cannot be obtained without radical restructuring of the theoretical diagram of the process. The following qualitative considerations are convincing in this respect.

As established by the direct measurements of Jacobs and Shade [8], a thin layer of superheated liquid ($\delta \gtrsim 0.127$ mm, carbon tetrachloride) exists around a bubble which has separated from the heating surface. The "surplus" heat content of this superheated layer provides primarily for the growth of the vapor bubble. But this is a relaxation process in which no constant factor exists to maintain the deviation from equilibrium. It is natural to assume that dynamic relaxation must take place most rapidly under

I. V. Kurchatov Institute of Atomic Energy, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 3, pp. 416-420, September, 1974. Original article submitted January 23, 1974.

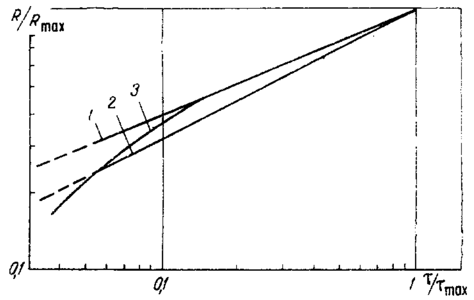

Fig. 1

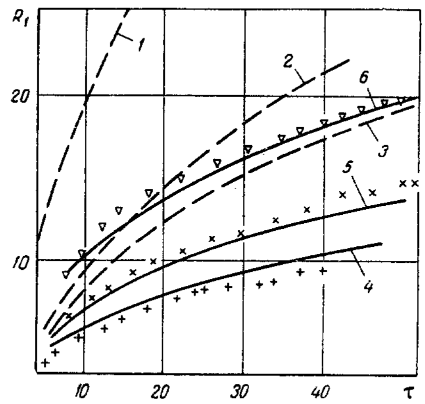

Fig. 2

Fig. 1. Time dependence of bubble radius: 1) data of [4]; 2) calculation from Eq. (1); 3) calculation from Eq. (6).

Fig. 2. Comparison of theoretical and experimental results ($\vartheta_W = 15°$; $p_\infty = 98$ torr, water): 1) calculation from an equation of the type (1) [6]; 2) calculation from an equation of the type (1) [5]; 3) calculation from (1); 4, 5, 6) calculation from (6) for points 4, 5, and 6, respectively. R, 10^{-3} m; τ, 10^{-3} sec.

these conditions and the evolution of the bubble will be determined by the condition $p_L = \text{idem} = p_\infty$ of uniformity of the pressure throughout the liquid. This is the formulation of the alternative causal diagram of the process under consideration.

The assumption of uniformity of the pressure throughout the liquid makes it possible to immediately draw several conclusions.

The pressure of the vapor in the bubble is

$$p_v = p_L + \frac{2\sigma}{R} = p_\infty + \frac{2\sigma}{R},$$

and since by definition p_∞ is the pressure of the saturated liquid, the bubble must be superheated by the value

$$\vartheta = T_v - T_s \approx \frac{2\sigma T_s}{L\rho_v R}. \tag{2}$$

Since $R(\tau)$ is determined by the condition $p_L = \text{idem} = p_\infty$, the heat flux from the superheated layer into the bubble (the flux of evaporation) is

$$-\lambda \frac{\partial T}{\partial R} = L\rho_v \dot{R}, \tag{3}$$

and consequently the effective thickness of the thermal boundary layer around the bubble will also be determined by the indicated dynamic condition.

The upper limit of this stage of bubble growth (R_{max}, τ_{max}) will be determined by the surplus heat content in the superheated layer of liquid, i.e.,

$$R_{max}^3 \lesssim \frac{\rho_L c_L \vartheta_w}{\rho_v L} \delta_0 R_0^2, \tag{4}$$

where R_0 has a value on the order of the separation size while the thickness δ of the superheated boundary layer around the bubble is

$$\delta_0 \sim \frac{\lambda \vartheta_w}{L\rho_v \dot{R}_0}. \tag{5}$$

Thus, with the condition $p_L = \text{idem} = p_\infty$ the Rayleigh equation takes the form

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{p_v-p_\infty}{\rho_L}-\frac{2\sigma}{\rho_L R}=\frac{p_L-p_\infty}{\rho_L}=0,$$

while its first integral is

$$R^3\dot{R}^2=\mu.$$

Finally,

$$R^{2.5}-R^{2.5}_{\tau=0}=2.5\mu^{0.5}\tau.$$

Usually in the asymptotic stage of growth one can neglect $R_{\tau=0}$ in comparison with R, so that

$$R=(2.5)^{0.4}\mu^{0.2}\tau^{0.4}. \tag{6}$$

For the determination of the integration constant μ it is required that R and $\dot{R}$ be given at an arbitrary time, which according to (2) and (3) is equivalent to giving the temperature and its gradient at the surface of the bubble.

Thus, with our causal diagram the theory gives an additional parameter of the initial state providing for the different values of $R(\tau)$ for single values of the initial superheat Ja. This result is quite regular from the point of view of the theory of activation of centers of vapor formation at heating surfaces, from which it follows that different values of R_0 correspond to different centers of vapor formation (nuclei) at the same values of ϑ_W.

A comparison with (1) shows that Eq. (6) gives slower bubble growth.

Data of [4] are presented in Fig. 1 in the form of the dependence of R/R_1 on τ/τ_1 (R_1 and τ_1 are the maximum experimental values of R and τ in [4]).

For long bubble lifetime $\tau \gtrsim 10^{-2}$ sec the growth of the latter is described by Eq. (6). For smaller values of τ the exponent n increases and apparently $n \to 1$ as $\tau \to 0$, which corresponds to the Rayleigh stage of bubble growth.

In Fig. 2 we present calculated values of $R(\tau)$ obtained from equations of the type (1) (curves 1, 2, 3) and from Eq. (6) (curves 4, 5, 6 for the corresponding cases).

Since R and $\dot{R}$ must be given at a certain time for the calculation of μ, these values were taken from experiment. It is not possible to calculate μ from the initial heat exchange parameters since the necessary information is absent from [4].

In conclusion let us estimate the upper limit of the stage of bubble growth under consideration for the experimental conditions of [4] ($\vartheta_W = 15°$, $p_\infty = 98$ torr).

The thickness of the layer of superheated liquid [Eq. (5)] surrounding the bubble after its separation from the wall is $\delta \approx 2\cdot 10^{-4}$ m (it is interesting to compare this with the data of Jacobs and Shade [8]).

The upper limit of this stage of growth [Eq. (4)] is determined by the values $R_{max} \approx 2\cdot 10^{-2}$ m and $\tau_{max} \approx 5.6\cdot 10^{-2}$ sec. The experimental values of [4] do not exceed the limits of this region.

NOTATION

R, radius of bubble; τ, time; σ, coefficient of surface tension; ρ_V, vapor density; ρ_L, liquid density; λ, thermal conductivity coefficient of liquid; ϑ, superheat of bubble; ϑ_W, superheat of wall; L, latent heat of evaporation; Ja, Jacob number; a_L, thermal diffusivity coefficient; T_S, saturation temperature; p_∞, pressure in bulk of liquid.

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